

Acoustical and optical radiation pressures and the development of single beam acoustical tweezers

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Abstract

Studies on radiation pressure in acoustics and optics have enriched one another and have a long common history. Acoustic radiation pressure is used for metrology, levitation, particle trapping and actuation. However, the dexterity and selectivity of single-beam optical tweezers are still to be matched with acoustical devices. Optical tweezers can trap, move and positioned micron size particles, biological samples or even atoms with subnanometer accuracy in three dimensions. One limitation of optical tweezers is the weak force that can be applied without thermal damage due to optical absorption. Acoustical tweezers overcome this limitation since the radiation pressure scales as the field intensity divided by the speed of propagation of the wave. However, the feasibility of single beam acoustical tweezers was demonstrated only recently. In this paper, we propose a historical review of the strong similarities but also the specificities of acoustical and optical radiation pressures, from the expression of the force to the development of single-beam acoustical tweezers.

Keywords:

Radiation pressure in optics, Radiation pressure in acoustics, Acoustical tweezers, Pseudo-momentum

1. Introduction

2 Radiation pressure is a mean force exerted by a wave that, in many situ-
3 ations, pushes an interface or a particle in the direction of propagation of the

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4 wave. This is usually interpreted as a transfer of linear momentum when the
5 wave is scattered. Since the wave at linear order is seen as an oscillation with
6 no mean momentum, radiation pressure is a nonlinear effect. It is common
7 to track back the beginning of the long and complex history of radiation
8 pressure to Kepler in 1619. On observing the specific shape of a comet's tail,
9 he made the assumption that the radiation from the sun exerts a force on
10 the comet's tail which changes its shape. The theoretical formulation of this
11 hypothesis was made more than two centuries later by Maxwell, who intro-
12 duced the stress tensor due to electromagnetic waves [1]. Hence, it is possible
13 to compute the magnitude of the so-called *radiation pressure* exerted by the
14 EM waves which is proportional to the ratio between the energy flux and
15 the speed of the light. Therefore, the magnitude of this second order effect
16 is very weak and the experimental demonstration of this force remained a
17 real challenge for a quarter of a century. At the beginning of the twentieth
18 century, two different experiments proposed by Lebedev [2], and Nichols and
19 Hull [3] validated Maxwell's theory. On the acoustics side, the story began,
20 as often, with the pioneering works of Lord Rayleigh [4, 5] who introduced
21 the counterpart of the EM radiation pressure for acoustic waves. Shortly
22 after Rayleigh's first paper and considering the experimental observations
23 made by Dvorak on acoustic fountains [6], Altberg [7] proposed to use the
24 radiation pressure to measure the amplitude of ultrasonic waves.

25 The experimental observation of the radiation pressure for electromag-
26 netic and acoustic waves was a real challenge at this early stage. As men-
27 tioned above, radiation pressure is due to second order effect. But many
28 phenomena may lead to second order effect and can contribute to an appar-
29 ent force and to erroneous interpretation. In optics, Crooks' radiometer was
30 claimed to be sensitive to radiation pressure. In fact, the force acting on
31 Crooks' radiometer was due to temperature gradient and its magnitude was
32 larger than the one expected. Nichols and Hull [3] improved the radiome-
33 ter to avoid this effect and succeeded in measuring the radiation pressure
34 with an amplitude in agreement with Maxwell's prediction. The situation
35 is different in acoustics. Indeed, while non adiabatic behavior is common
36 for gas, it is negligible for most liquids. However, wave attenuation occurs
37 also due to viscosity. This is a transfer of a kind of linear momentum from
38 the wave to the medium which generates a flow called acoustic streaming.
39 This effect is small but grows with the propagation distance and time. It
40 has to be distinguished from the radiation force, even though the distinction
41 is not always simple [8]. Even though the energy is conserved, the situation

42 remains subtle since the definition of mean linear momentum is quite difficult
43 as soon as the wave propagates in a material medium. Indeed the momentum
44 can be split in the wave momentum and the medium momentum and this
45 is somewhat arbitrary. This leads to the controversy between Minkowsky
46 and Abraham's momentum in optics. In acoustics, this also lead to many
47 difficulties, and many papers are devoted to the concept of momentum and
48 pseudo-momentum [9], [10]. Another difficulty, mostly in acoustics, comes
49 from the different sources of nonlinearities (nonlinear terms appear both in
50 the equation of momentum conservation and in the state equation). Com-
51 bined with the notion of linear momentum for acoustic waves, this is at the
52 core of the differences between Rayleigh and Langevin radiation pressure.
53 Indeed, in acoustics Rayleigh and Langevin proposed two definitions of the
54 radiation pressure due to a plane wave acting on an interface. This leads
55 to different theoretical expressions of the radiation pressure, the Rayleigh's
56 radiation pressure is sensitive to the nonlinear parameter of the medium,
57 while Langevin's is not. Brillouin proposed a different approach which is not
58 restricted to incident plane waves and he introduced a stress tensor which
59 has many similarities with the Maxwell's stress tensor [11].

60 Radiation pressure offers the ability to apply forces without physical both
61 in acoustics and in optics.

62 In acoustics, levitation traps have been known for a long time [12], [13],
63 [14]. Levitation traps are generally based on standing waves. For particles
64 which are very small in comparison with the wavelength, Gorkov proposed an
65 elegant theory. He derived a theoretical formulation of the radiation pressure
66 valid everywhere in an acoustic field due to standing waves. This formulation
67 is widely used because many acoustics applications meet Gorkov's assump-
68 tions. For instance, acoustic radiation pressure is widely used in microfluidics
69 to act on particles carried by the flow, it is called acoustophoresis [15, 16].
70 Even if 3D manipulations are possible, these kinds of devices do not allow
71 a selective control of a single particle [17, 18] because of the nature of the
72 acoustic field. Indeed, standing waves possess a lot of nodes and maxima
73 where the particles can be trapped in cluster. Despite its success and the
74 increasing number of applications based on this approach, it is probably not
75 the best one to develop selective traps, ie tweezers.

76 Solution was found in optics three decades ago by Ashkin who experi-
77 mentally demonstrated the possibility to trap a single dielectric particle with
78 a single-beam gradient force with a system called Optical Tweezers (OT in
79 short). The first step which paves the way to the OT is the observation by

80 Ashkin of the axial acceleration of particles illuminated by a laser beam and
81 the presence of a transverse force which attracts the particles toward the
82 beam axis [19]. This transverse force is now known as gradient force. The
83 second step was proposed in the same paper [19]. In order to make an axial
84 trap, Ashkin proposed to use a second laser sending a beam, whose prop-
85 agation direction is opposed to the first one. Hence, the lateral forces are
86 added while the axial ones subtract and do not engender an axial expelling.
87 Particles are trapped. From this seminal work, the development of OT took
88 sixteen years. In 1986, Ashkin *et al.* [20] proposed a new setup based on a
89 sharply focused laser beam able to exert a negative pulling force on a particle
90 located downstream from the focus. Hence, he demonstrated that a stable
91 equilibrium position exists and that it is possible to trap a single dielectric
92 particle with a single-beam gradient force. The negative force is due to a
93 subtle effect of the back scattering field on the particle involving the physical
94 properties of the particle and the incoming beam of light. The dexterity and
95 selectivity of optical tweezers is significantly superior to others optical traps
96 schemes and most applications of optical radiation pressure are made with
97 optical tweezers.

98 In acoustics, soon after, Du and Wu suggested theoretically to use ul-
99 trasonic beams to trap and manipulate small elastic particles[21]. However,
100 their analysis derived from Gorkov's theory confirmed that every solid elas-
101 tic particle was expelled from the intensity maxima by the gradient force.
102 Surprisingly in Wu's forthcoming experiment, the axial trapping failure was
103 explained by the only presence of acoustic streaming [22]. Using a coun-
104 terpropagating wave this axial expelling was canceled and trapping was ob-
105 tained. This set-up is the acoustic equivalent of "all-optical light trap" [23].
106 More recently, two dimensional manipulation was achieved with a focused
107 wave when the axial expelling is stopped by a membrane[24]. In optics,
108 this scheme is coined "single beam traps" [23], [19]. In a series of papers
109 [25, 26, 27], we published the theory and the experimental observations of
110 the first acoustical tweezers. The key was the shape of the beam. Indeed, pre-
111 vious studies used plane waves, gaussian beams or focused beams which exert
112 a pushing force on any solid particles. After a careful analysis of the scatter-
113 ing problem [25], we proposed to use a singular beam, namely an acoustical
114 vortex, which is a beam with a zero amplitude on its center [26, 27]. Finally,
115 the experimental demonstration of the all-acoustical single beam tweezers
116 was achieved by combining all these elements [27]. Figure 1 illustrates the
117 concept of acoustical tweezers acting on a single elastic sphere. At the same

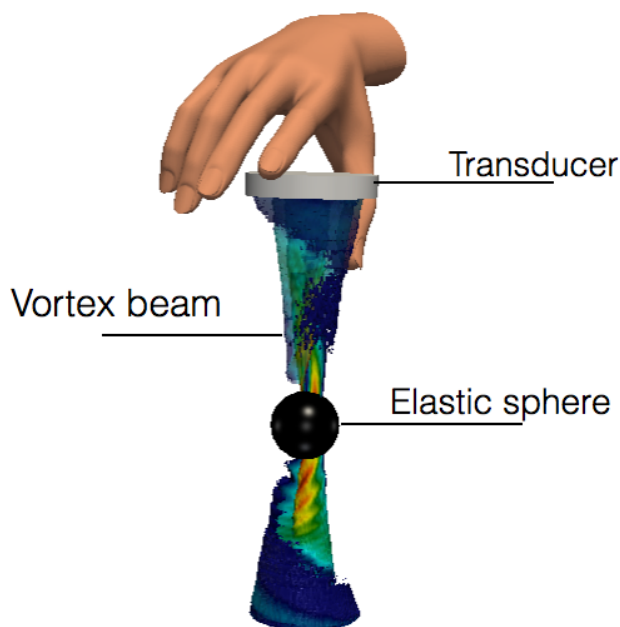


Figure 1: Schematic illustration of acoustical tweezers acting on a single elastic sphere. Note that, the size of the sphere is overstated to be visible

118 time another experimental demonstration was published [28]. The scheme
 119 are very similar, based on the same previous results, used the same kind of
 120 fields and the same kind of particles, polystyrene spheres. The main differ-
 121 ence is that, the trapping is carried in air. Thus the buoyancy force is quite
 122 weak and the weight of the particle precludes any direct demonstration of
 123 the axial pulling force.

124 The goal of this paper is to give the key concepts which make optical and
 125 acoustical tweezers fascinating devices able to control one particle remotely.
 126 Therefore, the radiation pressure due to EM or acoustical waves is presented
 127 in section 1. Special attention is paid to the derivation of Maxwell's stress
 128 tensor and its analogous Brillouin's stress tensor. These two tensors show
 129 that even though the radiation pressure is a nonlinear effect, it can be com-
 130 puted from the first order wave fields both in optics and acoustics. Therefore,
 131 solving the canonical problem of a sphere illuminated by an arbitrary shaped
 132 incident beam is mandatory. This is done thanks to the Generalized Lorenz-

133 Mie Theory in optics and its counterpart in acoustics, which are discussed
 134 in section 3. The GLMT provides a complete framework allowing to express
 135 the radiation pressure exerted on a sphere from the physical properties of the
 136 sphere and the parameter of the beam. The special case of small particles
 137 compared to the wavelength is discussed for acoustic waves in order to make
 138 bridges with usual formulations (Gorkov theory). Section 4 is devoted to
 139 the comparison of optical and acoustical tweezers. In particular, the choices
 140 associated to the properties of elastic particles and the incident beam are
 141 discussed in details for the acoustical case. Finally, a presentation of the
 142 advantages and the drawbacks of the two kinds of tweezers is proposed.

143 2. Radiation pressure and stress tensor

144 In this section, we review and compare the mathematical expression for
 145 radiation pressure in optics and in acoustics. We emphasize the strong sim-
 146 ilarities of the final expressions even though the derivation is different to
 147 accommodate the specificity of each field. At the end, we compare the op-
 148 tical and acoustical radiation pressures exerted by a plane wave and their
 149 relations with pseudo-momentum.

150 2.1. Radiation pressure tensor for an optical field

151 The first step consists in writing the equation of conservation of mechan-
 152 ical momentum and taking the average in time. The change in momentum
 153 of free charges, \mathbf{P}_{mech} , is related to the forces applied, i.e the Lorentz force:

$$\frac{\partial \mathbf{P}_{mech}}{\partial t} = \int_V (\rho \mathbf{E} + \mathbf{J} \wedge \mathbf{B}) dV. \quad (1)$$

154 First we consider a homogeneous medium : $\nabla \epsilon_{ij}^r = 0$ and $\nabla \mu_{ij}^r = 0$, where
 155 ϵ^r and μ^r are respectively the relative permittivity and the relative perme-
 156 ability. The latin subscripts (i, j) are used for spatial coordinates. From this
 157 equation and using Maxwell equations for a homogeneous medium without
 158 any electrostriction or magnetostriction, one can easily derive a continuity
 159 equation [29]:

$$\frac{\partial}{\partial t} \left(\mathbf{P}_{mech} + \int_V \mathbf{G}_M dV \right) + \int_V \nabla \cdot (-\mathcal{M}) dV = 0, \quad (2)$$

160 with

$$\mathcal{M}_{i,j} = E_i D_j + B_i H_j - 1/2 (E_k D_k + B_k H_k) \delta_{ij} \quad (3)$$

$$\mathbf{G}_M = \mathbf{D} \wedge \mathbf{B} = \mathbf{E} \wedge \mathbf{H}/c_n^2 = \mathbf{S}_o/c_n^2 \quad (4)$$

161 In these equations, c is the speed of light in vacuum, c_n the phase velocity
 162 of light in the medium of index n . \mathbf{G}_M is known as the Minkowski pseudo-
 163 momentum, \mathcal{M} is the Maxwell stress tensor and \mathbf{S}_o is the Poynting vector.
 164 If the medium considered contains no free charge carrier, $\mathbf{P}_{mech} = 0$, we ob-
 165 tain a continuity equation for the Minkowsky pseudo-momentum, \mathbf{G}_M [9],
 166 and $-\mathcal{M}$ is its flux. Note that the derived continuity equation, Eq. 2, is ob-
 167 tained with the assumption of a homogeneous medium. The general equation
 168 contains two terms proportional to $\nabla \epsilon_{ij}^r$ and $\nabla \mu_{ij}^r$, and is not a continuity
 169 equation. Hence Eq. 3 is based on the homogeneity of the medium, i.e in-
 170 variance with respect to a spatial translation of the material medium. This
 171 remark emphasizes the difference with the true linear momentum conserva-
 172 tion. Indeed, the linear momentum is related to the invariance with respect
 173 to translation of spatial coordinates [9].

174 The radiation pressure is a mean force, the next step is to take the time
 175 average, noted $\langle \rangle$, of Eq. 3 for a medium with no free charge. Note that for
 176 any stationary of periodic field, the mean of the derivative in time is equal
 177 to zero. This yields:

$$\nabla \cdot \langle \mathcal{M} \rangle = 0 \quad (5)$$

178 When there is an interface, for instance for a dielectric particle $\nabla \epsilon_{ij}^r \neq 0$,
 179 the conservation of $\langle \mathcal{M} \rangle$ is no longer true and a force is applied. This force
 180 is equal to the integral of the stress tensor on the surface of the particle, Σ :

$$\mathbf{F} = \int_S \langle \mathcal{M} \rangle \cdot \mathbf{n} dS \quad (6)$$

181 where \mathbf{n} is the unit vector normal to the surface element of the particle and
 182 pointing outward.

183 However using the conservation of the flux of pseudo-momentum, Eq. 5,
 184 and the theorem of divergence, the integral can be performed on any closed
 185 surface, S_R , outside the particle.

$$\mathbf{F} = \int_S \langle \mathcal{M} \rangle \cdot \mathbf{n} dS = \int_{S_R} \langle \mathcal{M} \rangle \cdot \mathbf{n}_R dS_R \quad (7)$$

186 One last remark, the definition of $\mathcal{M}_{i,j}$, Eq. 3, shows that all terms are
 187 quadratic quantities. This yields that this tensor can be computed at order
 188 2 with the linear fields.

189 *2.2. Radiation pressure tensor for an acoustical field*

190 In optics we have obtained a stress tensor, the Maxwell stress tensor, and
 191 identified the radiation pressure as the average of the stress applied on the
 192 particle surface. The radiation pressure is of second order since all terms
 193 appearing in the expression of the Maxwell stress tensor are quadratic with
 194 respect to the electric and magnetic fields. In acoustics, the stress tensor is
 195 well identified and, the force on the object is by definition the integral of the
 196 stress tensor on the surface of the particle. For a fluid the stress tensor σ_{ij}
 197 reduces to the pressure $-P\delta_{ij}$. However an acoustic wave is a displacement of
 198 the material particles of the medium of propagation. Therefore the surface of
 199 the object is moving and the amplitude of this displacement is proportional to
 200 the acoustic field. We can conclude that, as in the optical case, this quantity
 201 is at least of second order:

$$\mathbf{F} = - \int_{S(t)} P \mathbf{n} dS \quad (8)$$

202 and the radiation pressure is the mean component of this force

$$\langle \mathbf{F} \rangle = - \left\langle \int_{S(t)} P \mathbf{n} dS \right\rangle \quad (9)$$

203 To get an expression more tractable and make an analogy with optical ra-
 204 diation pressure, the fixed Euler coordinates are more convenient. The two
 205 points of view, Lagrange and Euler coordinates, are equivalent [30]. The ten-
 206 sorial theory of radiation pressure in Euler coordinates was established by
 207 Brillouin in a series of paper and an account of this contribution and these
 208 references can be found in his text book, [11]. As in optics, the first step is
 209 to write the continuity equation for momentum:

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \mathcal{B} = 0 \quad (10)$$

$$\mathcal{B}_{ij} = \rho v_i v_j + P \delta_{ij} \quad (11)$$

210 where ρ is the specific mass and \mathbf{v} the particle velocity. This equation can be
 211 integrated in a volume bounded on one side by the vibrating surface of the

212 particle and on the other side by a fixed surface in the fluid medium, Fig.
 213 2.2. Using the divergence theorem, we get:

$$\int_{V(t)} \frac{\partial \rho \mathbf{v}}{\partial t} dV + \int_{S(t)} \mathcal{B} \cdot \mathbf{n}' dS + \int_{S_R} \mathcal{B} \cdot \mathbf{n}_R dS_R = 0 \quad (12)$$

214 \mathbf{n}' is outward-pointing with respect to $V(t)$ and hence is the opposite of \mathbf{n}
 215 defined in Eq. 9. The first term would cancel with time averaging if we could
 216 commute the integral volume and the derivative in time. This mathematical
 217 step is the Reynolds transport theorem:

$$\frac{\partial}{\partial t} \int_{V(t)} \rho \mathbf{v} dV = \int_{V(t)} \frac{\partial \rho \mathbf{v}}{\partial t} dV + \int_{S(t)} \rho \mathbf{v} (\mathbf{v} \cdot \mathbf{n}' dS) \quad (13)$$

218 Using this theorem, the continuity relation can be rewritten:

$$\frac{\partial}{\partial t} \int_{V(t)} \rho \mathbf{v} dV + \int_{S(t)} P \mathbf{n}' dS + \int_{S_R} \mathcal{B} \cdot \mathbf{n}_R dS_R = 0 \quad (14)$$

219 We can now take the average in time and as in the optical case use the fact
 220 that the mean of a time derivative cancels:

$$\mathbf{F} = -\langle \int_{S(t)} P \mathbf{n} dS \rangle = -\int_{S_R} \langle \mathcal{B} \rangle \cdot \mathbf{n}_R dS_R \quad (15)$$

221 This expression is already comparable to the optical case, Eq. 7, the minus
 222 sign comes from the tensor used in acoustics, the flux of momentum, rather
 223 than a stress tensor. The radiation pressure is the integral of the flux of mo-
 224 mentum on a closed surface that delineates a volume containing the particle.
 225 To get this we needed to take into account the first specificity of acoustics :
 226 the surface of the object is vibrating due to the presence of the acoustic field.
 227 Since the radiation pressure is a second order effect, this vibration, while of
 228 weak amplitude, is not negligible. The second specificity is that we don't
 229 directly get a quadratic expression of the linear fields. To proceed further,
 230 we need to perform a perturbative decomposition of the fields up to second
 231 order. Assuming no flow at rest, this yields at second order:

$$\mathbf{v} = \mathbf{v}^1 + \mathbf{v}^2 \quad (16)$$

$$P = P^0 + P^1 + P^2 \quad (17)$$

$$\rho = \rho^0 + \rho^1 + \rho^2 \quad (18)$$

$$c_a = c^0 + c^1 + c^2 \quad (19)$$

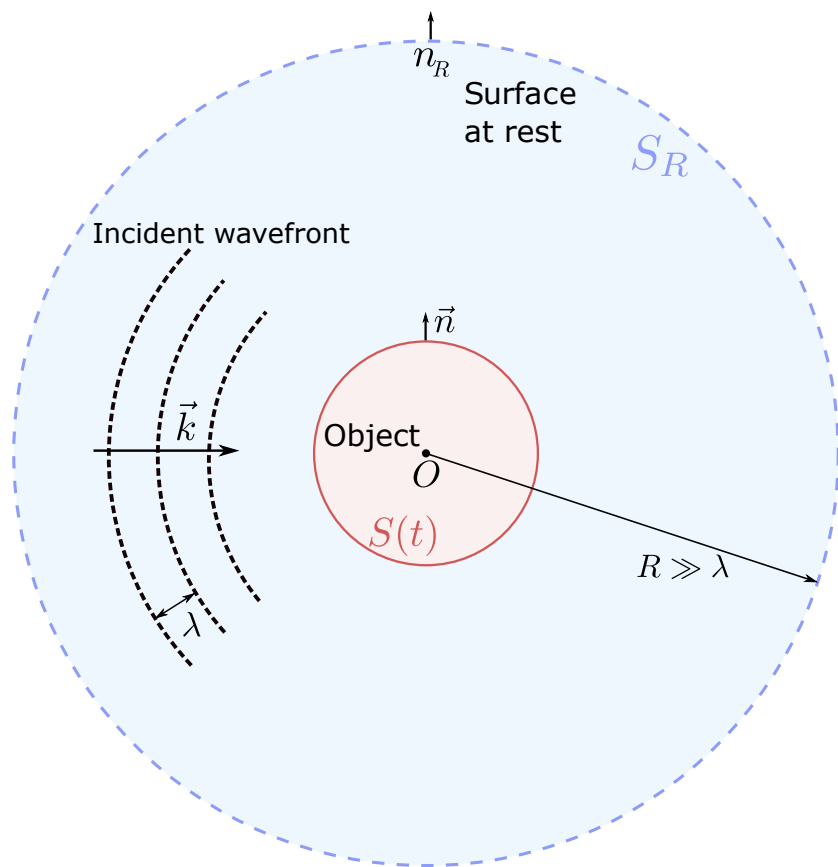


Figure 2: Scattering by a sphere.

232 where c_a is the speed of sound in the medium.

233 Many papers have been written to derive the equation giving the pressure
 234 at second order. This stage must be done very carefully and the boundary
 235 conditions must be taken into account [31], [32],[33]. For instance for a plane
 236 wave, i.e with an infinite lateral extension, or for a fluid which is laterally
 237 constrained by rigid wall, we get an expression that involves the non linearity
 238 of the state equation of the medium. This is the case studied by Rayleigh.
 239 However for practical cases, the radiation pressure is computed for an object
 240 embedded in a fluid or the wave is of limited extension. Contrary to the
 241 Rayleigh radiation pressure, here a static strain can relax laterally. This is
 242 the case presented in this review and the expression for the pressure at second
 243 order does not involve the nonlinear coefficient of the medium of propagation.
 244 These differences were first studied by Langevin and his analysis published
 245 by Biquard [34, 35].

$$P_E^2 = \frac{1}{2} \left(\frac{1}{\rho^0} \left(\frac{P^1}{c^0} \right)^2 - \rho^0 (\bar{v}^1)^2 \right) \quad (20)$$

246

$$\mathcal{B}_{ij} = \rho^0 v_i^1 v_j^1 + \frac{1}{2} \left(\frac{1}{\rho^0} \left(\frac{P^1}{c^0} \right)^2 - \rho^0 v_k^1 v_k^1 \right) \delta_{ij} \quad (21)$$

247 As for Maxwell's tensor, there is an isotropic term analog to a pressure and a
 248 tensorial term. However, the isotropic term is not the energy per unit volume
 249 yet. To ease the comparison with the optical case this last expression can be
 250 rewritten:

$$\mathcal{B}_{ij} = \frac{1}{2\rho^0} \left(\frac{P^1}{c^0} \right)^2 \delta_{ij} + \rho^0 v_i^1 v_j^1 - \frac{1}{2} \left(\frac{1}{\rho^0} \left(\frac{P^1}{c^0} \right)^2 + \rho^0 v_k^1 v_k^1 \right) \delta_{ij} \quad (22)$$

251 The two tensors \mathcal{B} and \mathcal{M} can now be compared. Both involve two fields \mathbf{E} , \mathbf{H}
 252 and p , \mathbf{v} respectively. The tensors split into an isotropic part and a tensorial
 253 part. The isotropic part is the energy density. The sum of the potential
 254 and kinetic energy in acoustics and the sum of the electric and magnetic
 255 energy in electromagnetism. These two energies yield two tensorial terms in
 256 optics. Acoustics gives an analog result but for a fluid medium the pressure
 257 field is a scalar. Note that the coefficient $1/(\rho c_0^2)$ is the bulk compressibility
 258 of the medium. However, one difference remains. For the optical case, we
 259 get the conservation of the pseudo-momentum while in acoustics we used

260 the conservation of momentum. We will see below that in the case studied
 261 here corresponding to the case derived by Langevin, this is also a pseudo-
 262 momentum. Here, we would like to emphasize one more time and after many
 263 papers on this subject that the momentum on an acoustic wave is null.

264 *2.3. The acoustic momentum and pseudo-momentum*

265 An acoustic wave is an oscillation of particles and no net flux is expected.
 266 Indeed, the momentum is

$$\rho\mathbf{v} = \rho^0\mathbf{v}^1 + \rho^1\mathbf{v}^1 + \rho^0\mathbf{v}^2 \quad (23)$$

267 If we take the time average, we get:

$$\langle\rho\mathbf{v}\rangle = \langle\rho^1\mathbf{v}^1\rangle + \rho^0\langle\mathbf{v}^2\rangle \quad (24)$$

268 So the mean mass flux is null if : $\langle\rho^1\mathbf{v}^1\rangle = -\rho^0\langle\mathbf{v}^2\rangle$. It can be demonstrated
 269 that this is the case for a plane wave, [36]. Counter-intuitively, $\langle\mathbf{v}^2\rangle$ points
 270 toward the source of sound. This quantity is a Eulerian quantity and rep-
 271 represents a physical quantity in a fixed position in space and not a quantity
 272 related to a given material particle. The mean velocity in Lagrangian coor-
 273 dinates, i.e. the mean velocity of material particles, is null as expected. $\langle\mathbf{v}^2\rangle$
 274 appears when the change of coordinates is performed and is the opposite of
 275 the Stokes drift. Using the state equations at first order, $p^1 = (c^0)^2\rho^1$, we
 276 get:

$$\rho^1\mathbf{v}^1 = P^1\mathbf{v}^1/(c^0)^2 = \mathbf{S}_a/(c^0)^2 \quad (25)$$

277 So while the total momentum is null, there is a finite pseudo-momentum
 278 that can be written as a quadratic expression of linear fields. The pseudo-
 279 momentum is equal to the acoustic Poynting vector, \mathbf{S}_a , divided by the square
 280 of the phase velocity in the medium. This is the relation obtained above with
 281 Minkowsky pseudo-momentum in the optical case, Eq. 4. In the next section
 282 we will see that the radiation pressure is related to this pseudo-momentum.

283 *2.4. Acoustical pseudo-momentum for a plane wave*

284 Let us consider a plane wave propagating along z and hence $\mathbf{v}^1 = (0, 0, v^1)$.
 285 The flux of momentum is :

$$\mathcal{B} = \rho^0(v^1)^2\delta_{zz} + \frac{1}{2} \left(\frac{1}{\rho^0} \left(\frac{P^1}{c^0} \right)^2 - \rho_0(v^1)^2 \right) \delta_{ij} \quad (26)$$

286

$$\langle \mathcal{B} \rangle \cdot \mathbf{z} = \langle E \rangle = \left\langle \frac{S_a}{c^0} \right\rangle = \langle c^0 \rho^1 v^1 \rangle \quad (27)$$

287 For this simple case, the radiation pressure is equal to the energy density,
 288 the Poynting vector divided by the phase velocity, or the pseudo-momentum
 289 times the phase velocity.

290 2.5. Optical pseudo-momentum for a plane wave

291 We consider a plane wave propagation along \mathbf{z} and hence $\mathbf{E} = (E, 0, 0)$
 292 and $\mathbf{B} = (0, B, 0)$. The Maxwell stress tensor is:

$$\mathcal{M} = \epsilon E^2 \delta_{xx} + \frac{1}{\mu} B^2 \delta_{yy} - \frac{1}{2} \left(\epsilon E^2 + \frac{B^2}{\mu} \right) \delta_{ij} \quad (28)$$

293 The component of the flux of pseudo-momentum along the direction of prop-
 294 agation, \mathbf{z} is :

$$-\langle \mathcal{M} \rangle \cdot \mathbf{z} = \langle E \rangle = \left\langle \frac{S_o}{c_n} \right\rangle = \langle c_n G_M \rangle \quad (29)$$

295 The optical case is identical to the acoustical case but the mean of the
 296 Minkowski pseudo-momentum replaces the mean of the acoustic pseudo-
 297 momentum, $\rho_1 v_1$.

298 3. Radiation force on a sphere exerted by an arbitrary shaped in- 299 cident beam

300 As long as the free-field condition is fulfilled, Langevin's expression of the
 301 excess of pressure can be used and the force is computed using Brillouin's
 302 stress or pseudo-momentum tensor. The latter only involves quadratic ex-
 303 pressions of the first order field as recalled in the previous section. Hence,
 304 one needs to calculate the total linear field (the sum of incident and reflected
 305 waves) and then to compute the force from these quantities. In this paper we
 306 pay attention interaction of waves with spheres as it is a very important case.
 307 First of all, the Lorenz-Mie theory is presented. This theory holds for inci-
 308 dent plane waves on a sphere. When the incident beam is not plane, different
 309 strategies have been developed. Here we focus on the Generalized Lorenz-Mie
 310 theory and in particular its extension to acoustics radiation problems which
 311 permits to compute the scattered field for an arbitrary beam. Then, consid-
 312 erations on the beam shape coefficients is proposed. Finally, the regime for
 313 which the size of the particle is very small compared to the wavelength is
 314 studied both in acoustics and optics.

315 *3.1. Scattering by a plane wave by a sphere*

316 The first derivation of the scattering of a plane wave by an elastic sphere in
317 an inviscid fluid was developed by Faran [37]. Interested in the propagation of
318 ultrasound through suspensions and emulsions like aerosols or other diphasic
319 solutions an important contribution was made by Epstein and Carhart [38]
320 that dealt with the scattering of a thermo-viscous fluid droplet suspended
321 in a thermo-viscous fluid. Allegra and Hawley generalized this model to the
322 case of a visco-elastic sphere [39]. This model is commonly referred to as
323 the ECAH model in acoustics. Importantly, an elastic medium supports the
324 existence of a one compressional and two transverse shear waves. Mathemat-
325 ically the displacement vector can be decomposed into its irrotational and
326 solenoidal parts using Helmholtz decomposition. The latter is the curl of a
327 potential vector \mathbf{A} . As in electromagnetism, this decomposition allows some
328 freedom and the vector potential \mathbf{A} satisfies the gauge condition $\nabla \cdot \mathbf{A} = 0$.
329 An interesting analogy lies in that this acoustic potential \mathbf{A} and both the
330 electric and magnetic fields \mathbf{E} , \mathbf{B} are solutions of the vector Helmholtz equa-
331 tion. Consequently, the same mathematical steps can be followed to retrieve
332 the solution. Nevertheless, the early developments in acoustics all consid-
333 ered the restricted case of a plane longitudinal wave incident on a spherical
334 scatterer [37, 38, 39, 40]. Owing to this strong symmetry condition the vector
335 potential in spherical coordinates (r, θ, ϕ) can be written $\mathbf{A} = (0, 0, A(r, \theta))$
336 for a plane wave propagating along the z axis. The problem is thereafter
337 considerably simplified inside the sphere and the scalar potential A is solu-
338 tion of the scalar Helmholtz equation. It can be written as an infinite sum of
339 spherical modes. The incident plane wave is expanded on the same spherical
340 modes and applying the correct boundary conditions yields the expression of
341 the unknown scattering coefficients necessary to calculate the total external
342 field. One can thereafter evaluate the force exerted on a compressible sphere
343 [41], [42].

344 It is noteworthy that a transversely polarized wave immediately breaks
345 the aforementioned azimuthal symmetry. It is the case for the scattering
346 of electromagnetic waves and the modern solution often referred to as the
347 Mie or Lorenz-Mie theory can actually be traced back to Clebsch and his
348 solution for the scattering of elastic waves by a rigid sphere [43, 44]. There are
349 various examples of the resolution of this type of boundary-value problem in
350 acoustics *e.g.* [45], [46]. A computationally useful method based on the "T-
351 Matrix" [47] was initially introduced in acoustics by Waterman [48, 49]. This
352 matrix does not depend on the nature of the incident wave. It is completely

353 defined by the scatterer and the propagation medium.

354 *3.2. Scattering of an arbitrarily shaped beam by a sphere*

355 With the growing interest in contact-less particle manipulation, recent
356 research has extended these approaches to the case of a particle located on the
357 axis of an incident acoustic beam. Examples include axisymmetric beams [50,
358 51] or the more complex wavefronts of a helicoidal bessel beam [52, 53]. To
359 deal with the scattering problem, the Bessel beam was decomposed as a series
360 of plane waves and the use of the previously cited results are straightforward.
361 Using an angular spectrum decomposition of the incident field, this result was
362 generalized to the case of a beam with arbitrary wavefront and the radiation
363 pressure was computed in three dimensions [54].

364 In optics, an important approach referred to as the Generalized Lorenz-
365 Mie Theory (GLMT) was developed [55, 56, 57], [58]. In the GLMT, the
366 problem under consideration is the scattering of an arbitrary incident wave
367 by an arbitrarily located dielectric particle. This model was successful in
368 obtaining the radiation pressure exerted by a beam of arbitrary wavefront
369 regardless on the relative position of the sphere [59], [60]. This efficient
370 approach was recently adapted to acoustics [25]. The incident wave is de-
371 composed in the spherical basis centered on the sphere using beam shape
372 coefficients. Note that even for a compressional incident acoustic wave, con-
373 sidering an arbitrary wavefront or an arbitrary location of the particle breaks
374 down the azimuthal symmetry $\mathbf{A} = (0, 0, A(r, \theta))$. Hence, as long as a spher-
375 ical basis is considered, a set of Debye scalar potentials (ψ, χ) for which
376 $\mathbf{A} = \nabla \wedge (\mathbf{r}\psi) + \nabla \wedge \nabla \wedge (\mathbf{r}\chi)$ is used to solve the vectorial Helmholtz equa-
377 tion on \mathbf{A} . These potentials are solutions of the scalar Helmholtz equation
378 so that ψ and χ can also be decomposed in the spherical basis. Hence the
379 incident, scattered and elastic waves in the sphere are readily described by
380 four independent potentials decomposed in spherical modes. The boundary
381 value problem yields the unknown scattering coefficients that are shown to
382 be identical to the usual plane wave case [25].

383 At this stage, the total acoustic field can be computed and Eq.(15) yields
384 the force exerted on the center of the sphere. It essentially depends on the
385 material of the sphere and the fluid's properties through the scattering coef-
386 ficients on one side, and on the nature of the incident field and the position
387 of the sphere through the beam shape coefficients on the other. An inter-
388 ested reader can refer to [25] for further details on the derivation. Note that
389 a similar result was obtained independently [61]. However, the generalized

390 scattering problem was not addressed and the results restricted to rigid (no
 391 internal propagation) spheres.

392 The general treatment in Ref.[25] can be extended to account for various
 393 other physical effects. For example, the ECAH theory sets the necessary lin-
 394 ear equations and introduces the vectorial treatment for elastic, viscous and
 395 thermal waves each of which can follow the same decomposition (Helmholtz
 396 and Debye) into a set of scalar potentials. These physical effects have shown
 397 to have a major influence on the radiation force on small particles compared
 398 to the wavelength [62, 63, 64] and can be deduced from the ECAH theory
 399 [65, 66]. Other extensions to the case of elastic shells [67, 68] were proposed
 400 in the context of the development of sonar detection. These models can find
 401 applications in radiation force calculations [69, 70].

402 *3.3. Computation of the beam shape coefficients*

403 In the context of GLMT theories, a major task resides in the accurate
 404 description of the incident beam and its position relative to the center of the
 405 scatterer. The beam shape coefficients fulfill this task and many techniques
 406 exist to obtain them. Their review is outside the scope of the present paper
 407 and the interested reader can refer to [57, 71]. A numerically efficient im-
 408 plementation using rotation and addition theorems for spherical harmonics
 409 is available in optics [72] and was adapted to acoustics. Examples include
 410 helicoidal Bessel beams [25], focused axisymmetric and vortex beams [26].

411 *3.4. Acoustic radiation force in the long-wavelength limit*

412 When the spherical scatterer has a very small radius a compared to the
 413 incident wavelength, the radiation force can be considerably simplified. On
 414 the one hand, only two vibrational modes of the sphere are excited. The
 415 first one is an isotropic monopolar expansion mode. It occurs when the
 416 compressibility of the scatterer differs from that of the fluid. The second one
 417 has a dipolar radiation pattern and arises from the back and forth oscillation
 418 of the sphere when a contrast of density between the two phases exists. In
 419 the small sphere limit, a Taylor expansion of the spherical Bessel functions
 420 involved in the two first scattering coefficients yields two different acoustic
 421 contrast factors [27]:

$$\alpha_m = \alpha_m^0 / (1 + i \frac{k^3}{4\pi} \alpha_m^0) \quad (30)$$

$$\alpha_d = \alpha_d^0 / (1 - i \frac{k^3}{12\pi} \alpha_d^0) \quad (31)$$

422 where $k = \omega/c^0$ is the wave number in the liquid and

$$\alpha_m^0 = \frac{4}{3}\pi a^3 \left(1 - \frac{K^0}{K^p}\right) \quad (32)$$

$$\alpha_d^0 = 4\pi a^3 \left(\frac{\rho^p - \rho^0}{2\rho^p + \rho^0}\right). \quad (33)$$

423 $K^0 = \rho(c^0)^2$ is the bulk elasticity of the material in the fluid and $K^p =$
 424 $\rho^p(4/3c_t^2 - c_l^2)$ in the solid. The sphere's density, longitudinal and transverse
 425 wave speeds are noted ρ^p , c_l and c_t respectively.

426 On the other hand, the beam shape coefficients are formally scalar projec-
 427 tions of the incident field on the spherical basis. In the same long wavelength
 428 limit, the beam shape coefficients can be written as linear combinations of
 429 the derivatives of the incident field taken at the center of the sphere [27].
 430 The final expression of the force for a small elastic sphere in an inviscid fluid
 431 reads:

$$\begin{aligned} \mathbf{F} = & -\frac{1}{2} \left\{ \Re(\alpha_m) \nabla \left(\frac{1}{2\rho^0} \left(\frac{|P^1|}{c^0} \right)^2 \right) - \Re(\alpha_d) \nabla \left(\frac{1}{2}\rho^0 |\mathbf{v}^1|^2 \right) \right. \\ & + \left(\frac{k}{c^0} \Im(\alpha_m) - \frac{k^4}{12\pi c^0} \Re(\alpha_m) \Re(\alpha_d) \right) \Re(P^1 \mathbf{v}^{1*}) \\ & \left. + \rho^0 \Im(\alpha_d) \Im((\mathbf{v}^1 \cdot \nabla) \mathbf{v}^{1*}) \right\}. \end{aligned} \quad (34)$$

432 \Re and \Im denote the real and imaginary parts of these complex fields and *
 433 stands for a complex conjugations.

434 The first two terms in Eq.(34) stand for an acoustic gradient force. The
 435 real part of the monopolar scattering factor is associated with the gradient
 436 of the potential energy density of the field while the real part of the dipolar
 437 scattering factor is linked to the gradient of the kinetic energy density. It is
 438 a force proportional to the volume of the sphere a^3 . Gorkov was probably
 439 the first to show that the radiation pressure of a standing wave field could be
 440 written as a gradient force [73]. His result is here recovered. The remaining
 441 term is called the scattering force and is associated to the imaginary parts of
 442 the scattering coefficients and a coupling between the monopolar and dipolar
 443 modes. Gorkov had also shown that the force exerted by a plane progressive
 444 wave was much weaker since there are no gradients in the fields' energy
 445 density. Indeed the imaginary part of the monopolar and dipolar contrast

446 factors are multiplied by an additional factor $(ka)^3 \ll 1$. Eq. 34 generalizes
447 Gorkov's result to account for the scattering force of an arbitrary wavefield.
448 A similar result was obtained elsewhere [54].

449 It is known that a very slight viscosity in the fluid suffices to drastically
450 increase the magnitude of the scattering term of the force [62]. It is worth
451 noting that the two acoustic contrast factors in Eq.(31) can be modified
452 to account for the thermo-viscosity of the fluid or the visco-elasticity of the
453 sphere [64, 66, 65, 70].

454 *3.5. Long-wavelength simplified expression in optics and acoustics*

455 The optical radiation force on a small dielectric sphere has a very similar
456 expression [74],[75],[76]:

- 457 • There are also two modes but this time both are dipolar and related to
458 the contrast in dielectric permittivity and magnetic permeability. Note
459 that there is a single dipolar mode in acoustics because we assumed an
460 elastic sphere in a fluid medium. The radiation pressure on a spherical
461 inclusion in a solid would involve transverse incident waves.
- 462 • The real part of the scattering coefficient, the clausius-mossotti rela-
463 tion is identical to the acoustic dipolar scattering coefficients if specific
464 mass is replaced by either the dielectric permittivity or the magnetic
465 permeability.
- 466 • The scattering coefficients have a small imaginary part proportional to
467 the square of the real part. This correction was recently introduced in
468 optics [76].
- 469 • The force can be split in a gradient and a scattering force. In optics this
470 was inferred independently of the correct expression of the scattering
471 coefficients [75] and compared to the full theory provided by the GLMT.
- 472 • The gradient force is related to the energy density of the incident fields
473 at the sphere center and the real part of the scattering coefficient.
- 474 • The scattering force is weaker and related to the imaginary part of the
475 scattering coefficient. In both cases there is a term proportional to the
476 mean of the Poynting vector.

477 There are of course differences due to polarizations. In acoustics, the wave
478 is scalar and longitudinally polarized in the liquid and have longitudinal and
479 transverse components in the solid sphere. In optics the wave is transverse
480 in both media.

481 **4. Acoustical tweezers**

482 Applying controlled forces without contact is appealing. It has many
483 practical applications, both in optics and in acoustics. Optical tweezers can
484 manipulate sub-micrometric objects with a nanometer resolution and forces
485 in the pico-newton range. They have found a huge amount of applications
486 from fundamental physics to material science and biophysics [77, 78, 79, 80,
487 81]. Acoustic traps can considerably increase the size of the manipulated
488 particles and the force that can be exerted. From the first acoustic levitation
489 traps [13, 14] to the recent regain in interest in the context of acoustofluidics,
490 acoustic traps have addressed a significant panel of new applications [15,
491 82, 83]. In this section we propose to review the recent demonstration of
492 single-beam acoustical tweezers for which, in comparison to other traps, the
493 development has been rather slow.

494 Unlike optical trapping of high-index particles, solid elastic materials are
495 not transparent to ultrasound. The mechanical index mismatch is such that
496 the preeminent mechanism at the fluid/solid boundary is backscattering in-
497 stead of refraction. Moreover, considering a large sphere in the geometrical
498 acoustics regime ($a \ll \lambda$), not only the scattering force pushes the parti-
499 cle but, the refracted rays that in optics usually contribute to the restoring
500 gradient component build up to an additional expelling force in acoustics.
501 Restricting ourselves to much smaller spheres and avoiding as a first step the
502 more complex Mie scattering regime ($a \leq \lambda$), a quick analysis of Gorkovs
503 theory (see Eq.(15)) shows that a particle that is denser and stiffer than the
504 surrounding medium experiences a gradient force that points away from in-
505 tensity maxima. Hence, whatever the size of the solid particle, acoustic beam
506 traps generally exhibit an unstable behavior and the history of acoustic par-
507 ticle manipulation has almost always involved standing waves schemes.

508 Holding on to the single-beam concept, it was recently recognized that
509 specific fields called acoustic vortices could act as stable lateral [84] and three-
510 dimensional traps [26]. The lateral trapping of vortex type beams was con-
511 firmed experimentally in a planar configuration creating 2D annular Bessel
512 function shaped traps [85]. Using a three-dimensional theory for acoustic

513 forces [25] it was possible to compute the axial trapping force of tightly fo-
514 cused vortex beams [26]. It was concluded that a focused vortex beam of
515 topological charge $m = 1$ could generate a negative axial gradient force to
516 stably trap elastic particles in three dimensions. In fact, using sufficiently
517 small spheres ($a \leq 0.15\lambda$), the scattering force is greatly diminished on the
518 axis of a vortex beam meanwhile the gradient component points this time
519 towards the focus. Note that other types of beams were proposed in [26] to
520 enhance the axial trapping efficiency similar to what is referred to as "bottle
521 beams" in optics [86, 87].

522 The research field of structured wavefronts is again intimately related
523 between optics and acoustics. Though the seminal paper published by Nye
524 and Berry in 1974 first introduced the phenomenon of phase singularities
525 within ultrasonic wave trains [88], the optics community rapidly pushed the
526 concept forward towards a whole deal of fundamental studies and applica-
527 tions [89, 90, 91]. Remarkably, a route to directly create a beam carrying
528 a screw phase dislocation was first theoretically proposed in acoustics [92]
529 at the time it was recognized that TEM_{01}^* laser modes could be generated
530 [91, 93]. Experimentally demonstrated by Hefner and Marston [94]. Studies
531 of their linear and non-linear behavior include the establishment of a law
532 of conservation of their topological charge and pseudo-angular momentum
533 [94, 95, 96], vortex parametric interaction [97], azimuthal shock waves [98]
534 and their super-oscillation properties applied to sub-wavelength imaging [99].
535 An example of a synthesized vortex beam is given in Fig.3. The ultrasonic
536 field is generated by a 128 element piezoelectric array in a water tank using
537 the inverse filter technique [95]. As for its optical counterpart, the energy is
538 focused to a ring in the focal plane (panel a)). Note that a hydrophone deliv-
539 ers a direct measurement of the spiraling phase structure while in optics the
540 vortex beam generally has to interfere with a plane wave. It is noteworthy
541 that a high numerical aperture acoustic lens was designed to focus the beam
542 to a ring of diameter comparable to λ . The region of undefined phase is a
543 line in three dimensions. Consequently, the entire propagation axis defines a
544 silent zone (see Fig.3c)).

545 The control achieved in creating tightly focused vortex beams led us to
546 experimentally demonstrate the existence of a negative gradient force and in
547 essence observe the first single-beam gradient trap for elastic particles with an
548 ultrasonic beam [27]. In the first configuration a vortex was fired horizontally
549 in the water tank while a polystyrene particle was approached near the focal
550 region by an auxiliary tee. Figure 4 shows a photograph of a $400\mu\text{m}$ size

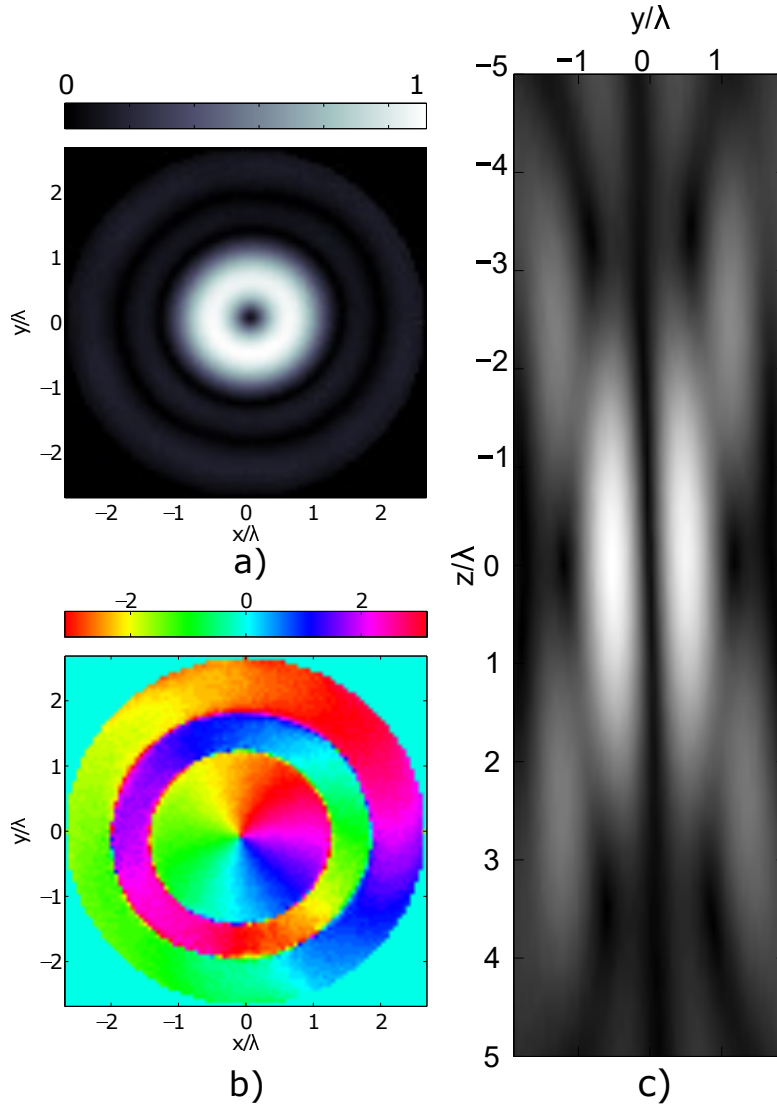


Figure 3: Example of a synthesized vortex beam of topological charge $m = 1$. a) and b), normalized intensity and phase (rad.) in the focal plane respectively. c), normalized intensity along the propagation axis. Adapted from [65].

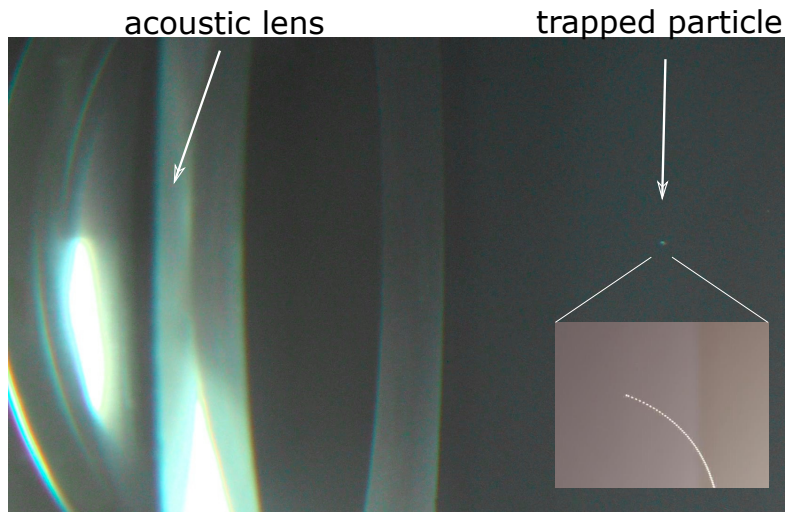


Figure 4: Photograph of a trapped polystyrene particle in a horizontal acoustical tweezers setup. A zoomed range of the particle's release is displaced with an overlap of images. The particle's trajectory shows the effect of gravity and acoustic streaming.

551 polystyrene sphere trapped in the focus of the helicoidal field. The particle
 552 levitates approximately 30mm away from the outer face of the lens as long
 553 as the vortex is emitted. When the source is turned off, an overlapped image
 554 stack of the release of the sphere is shown. It can be seen from the trajectory
 555 that at this scale that gravity plays a major role and remark the significant
 556 effect of acoustic streaming pushing the particle away from the focus.

557 It was decided to demonstrate that the negative gradient force was so
 558 large that it dominated the axial stability in a vertical configuration. The
 559 experimental setup described in [27] was used to lift and trap buoyant par-
 560 ticles against their weight and the pushing force exerted by the streaming
 561 flow's drag. Figure 5 is a photograph of a $340\mu\text{m}$ polystyrene bead trapped
 562 and levitated beneath the focus of the vortex beam. The bead was initially
 563 lying on an acoustically transparent polyethylene film. By precisely aim-
 564 ing the beam, the tweezers can accurately select the particle to be trapped.
 565 Other particles can be slightly affected but will not collect in the the focal
 566 volume.

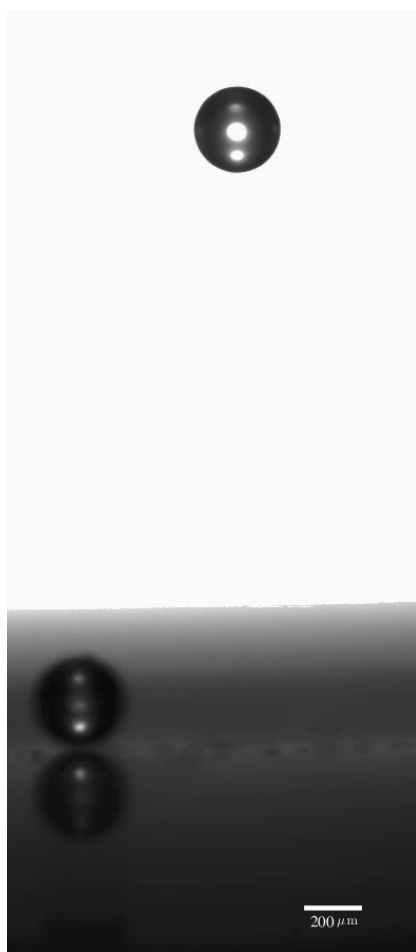


Figure 5: Photograph of a $340\mu\text{m}$ diameter polystyrene sphere trapped in vertical acoustical tweezers. The trapped particle was selected among others that were lying on a thin polyethylene film. Adapted from [27]

567 5. Conclusion

568 The radiation pressures exerted by sound or light have many similar fea-
569 tures. This was shown with the expression of the stress tensor and the
570 relation with pseudo-momentum. Not only these second order effects are
571 similar but also when the linear problem of a spherical scatterer interacting
572 with an incident beam is considered. The Generalized Lorenz-Mie theory
573 has therefore recently found an adaptation to acoustic scattering and force
574 calculations. Regarding the force in the long wavelength limit, it was shown
575 that a beam exerted both a gradient and scattering force in acoustics in a
576 similar fashion to the force exerted by an optical beam on a small dielec-
577 tric dipole. The possibility to design manipulation traps for small particles
578 is another appealing and common characteristic. Among all kinds of traps,
579 optical tweezers stand out by their simplicity, accurate localized actuation
580 and great dexterity. Quoting Ashkin [20]: "The single-beam gradient force
581 trap is conceptually and practically one of the simplest radiation pressure
582 traps". A feature which most certainly explains their wide application in
583 various scientific fields. The attention of the acoustic community has essen-
584 tially been turned towards standing wave traps where Gorkov's theory has
585 found a sound application. They have addressed a large panel of new appli-
586 cations in the context of acoustical levitation or acoustofluidics [15, 82, 83].
587 A recent review offers an extensive survey of various developed devices [100].
588 The development of single-beam acoustical tweezers had been impaired by
589 two main difficulties: the lack of a complete theoretical model able to predict
590 the force exerted by acoustic beams and the capacity to synthesize acoustic
591 beams with complex wavefronts as focused acoustical vortices. The complex
592 wavefield synthesis method with a large array of piezo-electric transducers
593 was adapted from previous studies on acoustical vortices [95, 96].

594 The thermal damage, or "opticutition", limit optical tweezers to applica-
595 tions requiring very weak forces and to manipulate very small particles from
596 atoms to molecules. Ultrasound are proven to innocuous for living cells and
597 propagate without significant attenuation in many materials. This feature
598 combined with the much larger forces applied at equivalent energy flux should
599 give to single-beam acoustical tweezers an extended range of manipulation
600 capacities for new applications in material science, fluidics and biophysics.

- 601 [1] J. C. Maxwell, Art. 314. medium in which small spheres are uniformly
602 disseminated, chapter ix. conduction through heterogeneous media, A
603 treatise on electricity and magnetism 1.

- 604 [2] P. Lebedev, Experimental examination of light pressure, *Annalen der*
605 *physik* 6 (1901) 433.
- 606 [3] E. F. Nichols, G. F. Hull, The pressure due to radiation, in: *Daedalus*,
607 Vol. 38, 1903, pp. 559–599.
- 608 [4] L. Rayleigh, *Phil. Mag.* 3 (1902) 338.
- 609 [5] L. Rayleigh, *Phil. Mag.* 10 (1905) 364.
- 610 [6] V. Dvorak, On acoustic repulsion, *Am. J. Sci.* 16 (1878) 22–29.
- 611 [7] W. Altberg, ber die druckkrfte der schallwellen und die absolute mes-
612 sung der schallintensitt, *Annalen der Physik* 316 (6) (1903) 405–420.
- 613 [8] C. Eckart, Vortices and streams caused by sound waves 73 (1) (1948)
614 68–76.
- 615 [9] R. Peirls, Momentum and pseudomomentum of light and sound, *Proc.*
616 *Intl. School Phys. "Enrico Fermi"*, Elsevier Science Ltd, 1985, pp. 237–
617 255.
- 618 [10] M. McIntyre, On the "wave momentum" myth, *J. Fluid. Mech.* 106
619 (1981) 331–347.
- 620 [11] L. Brillouin, *Tensors in mechanics and elasticity*, Academic Press, New
621 York, 1964.
- 622 [12] A. Eller, Force on a bubble in a standing acoustic wave, *J. Acoust. Soc.*
623 *Am.* 43 (1) (1968) 170–171.
- 624 [13] E. Trinh, Compact acoustic levitation device for studies in fluid dy-
625 namics and material science in the laboratory and microgravity, *Rev.*
626 *Sci. Instrum.* 56 (11) (1985) 2059–2065.
- 627 [14] R. E. Apfel, Acoustic levitation for studying liquids and biological ma-
628 terials, *J. Acoust. Soc. Am.* 70 (2) (1981) 636–639.
- 629 [15] T. Laurell, F. Petersson, A. Nilsson, Chip integrated strategies for
630 acoustic separation and manipulation of cells and particles, *Chem. Soc.*
631 *Rev.* 36 (3) (2007) 492–506.

- 632 [16] P. B. Muller, M. Rossi, Á. Marín, R. Barnkob, P. Augustsson, T. Lau-
633 rell, C. J. Kaehler, H. Bruus, Ultrasound-induced acoustophoretic mo-
634 tion of microparticles in three dimensions, *Phys. Rev. E* 88 (2) (2013)
635 023006.
- 636 [17] F. Guo, P. Li, J. B. French, Z. Mao, H. Zhao, S. Li, N. Nama, J. R.
637 Fick, S. J. Benkovic, T. J. Huang, Controlling cell–cell interactions
638 using surface acoustic waves, *Proc. Natl. Acad. Sci. USA* 112 (1) (2015)
639 43–48.
- 640 [18] C. R. Courtney, C. E. Demore, H. Wu, A. Grinenko, P. D. Wilcox,
641 S. Cochran, B. W. Drinkwater, Independent trapping and manipulation
642 of microparticles using dexterous acoustic tweezers, *App. Phys. Lett.*
643 104 (15) (2014) 154103.
- 644 [19] A. Ashkin, Acceleration and trapping of particles by radiation pressure,
645 *Phys. Rev. Lett.* 24 (4) (1970) 156–159.
- 646 [20] A. Ashkin, J. Dziedzic, J. Bjorkholm, S. Chu, Observation of a single-
647 beam gradient force optical trap for dielectric particles, *Optics letters*
648 11 (5) (1986) 288–290.
- 649 [21] J. Wu, G. Du, Acoustic radiation force on a small compressible sphere
650 in a focused beam, *J. Acoust. Soc. Am.* 87 (3) (1990) 997–1003.
- 651 [22] J. Wu, Acoustical tweezers, *J. Acoust. Soc. Am.* 89 (5) (1991) 2140–
652 2143.
- 653 [23] A. Ashkin, How it all began, *Nature Photonics* 5 (2011) 316–317.
- 654 [24] J. Lee, K. Shung, Radiation forces exerted on arbitrarily located sphere
655 by acoustic tweezer, *J. Acoust. Soc. Am.* 120 (2) (2006) 1084–1094.
- 656 [25] D. Baresch, J.-L. Thomas, R. Marchiano, Three-dimensional acoustic
657 radiation force on an arbitrarily located elastic sphere, *J. Acoust. Soc.*
658 *Am.* 133 (1) (2013) 25–36.
- 659 [26] D. Baresch, J.-L. Thomas, R. Marchiano, Spherical vortex beams of
660 high radial degree for enhanced single-beam tweezers, *J. Appl. Phys.*
661 113 (18) (2013) 184901.

- 662 [27] D. Baresch, J.-L. Thomas, R. Marchiano, Observation of a single-beam
663 gradient force acoustical trap for elastic particles: acoustical tweezers,
664 Phys. review lett. 116 (2) (2016) 024301.
- 665 [28] A. Marzo, S. A. Seah, B. W. Drinkwater, D. R. Sahoo, B. Long, S. Sub-
666 ramanian, Holographic acoustic elements for manipulation of levitated
667 objects, Nat. Commun. 6 (2015) 8661.
- 668 [29] J. Jackson, Classical Electrodynamics, John Wiley and Sons Inc., New
669 York, 1962, Ch. 6.
- 670 [30] K. Beissner, The acoustic radiation force in lossless fluids in eulerian
671 and lagrangian coordinates, J. Acoust. Soc. Am. 103 (5) (1998) 2321–
672 2332.
- 673 [31] E. J. Post, Radiation pressure and dispersion, J. Acoust. Soc. Am.
674 25 (1) (1953) 55–60.
- 675 [32] K. Beissner, Two concepts of acoustic radiation pressure, J. Acoust.
676 Soc. Am. 79 (5) (1986) 1610–1612.
- 677 [33] R. Beyer, Radiation pressure-the history of a mislabeled tensor, J.
678 Acous. Soc. Am. 63 (4) (1978) 1025–1030.
- 679 [34] P. Biquard, Rev. Acoust. 1 (1932) 93–109.
- 680 [35] P. Biquard, Rev. Acoust. 2 (1933) 315–335.
- 681 [36] B. Chu, R. E. Apfel, Acoustic radiation pressure produced by a beam
682 of sound, J. Acous. Soc. Am. 72 (6) (1982) 1673–1687.
- 683 [37] J. Faran, Sound scattering by solid cylinders and spheres, J. Acoust.
684 Soc. Am. 23 (1951) 405–418.
- 685 [38] P. Epstein, R. Carhart, The absorption of sound in suspensions and
686 emulsions. i. water fog in air., J. Acoust. Soc. Am. 25 (3) (1953) 553–
687 565.
- 688 [39] J. Allegra, S. Hawley, Attenuation of sound in suspensions and emul-
689 sions: Theory and experiments, J. Acoust. Soc. Am. 51 (5) (1971)
690 1545–1564.

- 691 [40] P. M. Morse, *Vibration and sound*, Vol. 2, McGraw-Hill New York,
692 1948.
- 693 [41] K. Yosioka, Y. Kawasima, Acoustic radiation pressure on a compress-
694 ible sphere, *Acustica* 5 (3) (1955) 167–173.
- 695 [42] T. Hasegawa, K. Yosioka, Acoustic radiation force on a solid elastic
696 sphere, *J. Acoust. Soc. Am.* 46 (5) (1969) 1139–1143.
- 697 [43] N. A. Logan, Early history of the mie solution, *J. Opt. Soc. Am.* 52 (3)
698 (1962) 342–343.
- 699 [44] N. A. Logan, Survey of some early studies of the scattering of plane
700 waves by a sphere, *Proc. IEEE* 53 (8) (1965) 773–785.
- 701 [45] N. Einspruch, E. Witterholt, R. Truell, Scattering of a plane transverse
702 wave by a spherical obstacle in an elastic medium, *J. Appl. Phys.* 31 (5)
703 (1960) 806–818.
- 704 [46] G. C. Gaunard, H. Überall, Theory of resonant scattering from spheri-
705 cal cavities in elastic and viscoelastic media, *J. Acoust. Soc. Am.* 63 (6)
706 (1978) 1699–1712.
- 707 [47] T. Nieminen, H. Rubinsztein-Dunlop, N. Heckenberg, Calculation of
708 the t-matrix: general considerations and application of the point-
709 matching method, *J. Quant. Spect. and Rad. Transf.* 7980 (2003) 1019
710 – 1029.
- 711 [48] P. Waterman, New formulation of acoustic scattering, *J. Acoust. Soc.*
712 *Am.* 45 (6) (1969) 1417–1429.
- 713 [49] P. Waterman, Matrix theory of elastic wave scattering, *J. Acoust. Soc.*
714 *Am.* 60 (3) (1976) 567–580.
- 715 [50] X. Chen, R. Apfel, Radiation force on a spherical object in the field
716 of a focused cylindrical transducer, *J. Acoust. Soc. Am.* 101 (5) (1996)
717 2443–2447.
- 718 [51] P. L. Marston, Axial radiation force of a bessel beam on a sphere and
719 direction reversal of the force, *J. Acoust. Soc. Am.* 120 (6) (2006) 3518.

- 720 [52] P. L. Marston, Scattering of a bessel beam by a sphere: li helicoidal
721 case and spherical shell example, *J. Acoust. Soc. Am.* 124 (5) (2008)
722 2905–2910.
- 723 [53] P. L. Marston, Radiation force of a helicoidal bessel beam on a sphere,
724 *J. Acoust. Soc. Am.* 120 (2009) 3539–3547.
- 725 [54] O. A. Sapozhnikov, M. R. Bailey, Radiation force of an arbitrary acous-
726 tic beam on an elastic sphere in a fluid, *J. Acoust. Soc. Am.* 133 (2)
727 (2013) 661–676.
- 728 [55] B. Maheu, G. Gouesbet, G. Gréhan, A concise presentation of the
729 generalized lorenz-mie theory for arbitrary location of the scatterer in
730 an arbitrary incident profile, *J. Opt.* 19 (2) (1987) 59–67.
- 731 [56] G. Gouesbet, B. Maheu, G. Gréhan, Light scattering from a sphere
732 arbitrarily located in a gaussian beam, using a bromwich formulation,
733 *JOSA A* 5 (9) (1988) 1427–1443.
- 734 [57] G. Gouesbet, J. Lock, G. Gréhan, Generalized lorenzmie theories and
735 description of electromagnetic arbitrary shaped beams : Localized ap-
736 proximations and localized beam models,a review, *J. Quant. Spectr. and*
737 *Rad. Transf.* 112 (2010) 1–27.
- 738 [58] J. P. Barton, D. R. Alexander, S. A. Schaub, Internal and near-surface
739 electromagnetic fields for a spherical particle irradiated by a focused
740 laser beam, *J. Appl. Phys.* 64 (4) (1988) 1632–1639.
- 741 [59] K. Ren, G. Gréhan, G. Gouesbet, Radiation pressure forces exerted on
742 a particle arbitrarily located in a gaussian beam by using the general-
743 ized lorenz-mie theory, and associated resonance effects, *Optics Comm.*
744 108 (1994) 343–354.
- 745 [60] J. P. Barton, D. R. Alexander, S. A. Schaub, Theoretical determination
746 of net radiation force and torque for a spherical particle illuminated by
747 a focused laser beam, *J. Appl. Phys.* 66 (10) 4594–4602.
- 748 [61] G. T. Silva, An expression for the radiation force exerted by an acoustic
749 beam with arbitrary wavefront, *J. Acoust. Soc. Am.* 130 (6) (2011)
750 3541–3544.

- 751 [62] S. Danilov, M. Mironov, Mean force on a small sphere in a sound field
752 in a viscous fluid, *J. Acoust. Soc. Am.* 107 (1) (2000) 143–153.
- 753 [63] A. A. Doinikov, Acoustic radiation pressure on a rigid sphere in a
754 viscous fluid, *Proc Royal Soc. London. A.* 447 (1994) 447–466.
- 755 [64] M. Settnes, H. Bruus, *Physical Review E* 85 (1) (2012) 016327.
- 756 [65] D. Baresch, Pince acoustique: piégeage et manipulation d’un objet
757 par pression de radiation d’une onde progressive, Ph.D. thesis, Paris 6
758 (2014).
- 759 [66] J. T. Karlsen, H. Bruus, Forces acting on a small particle in an acous-
760 tical field in a thermoviscous fluid, *Phys. Rev. E* 92 (2015) 043010.
- 761 [67] V. Ayres, G. C. Gaunard, Acoustic Resonance Scattering by Vis-
762 coelastic Objects, *J. Acoust. Soc. Am.* 81 (2).
- 763 [68] M. C. Junger, Sound scattering by thin elastic shells, *The Journal of*
764 *the Acoustical Society of America* 24 (4) (1952) 366–373.
- 765 [69] T. Hasegawa, Y. Hino, A. Annou, H. Noda, M. Kato, N. Inoue, Acous-
766 tic radiation pressure acting on spherical and cylindrical shells, *The*
767 *Journal of the Acoustical Society of America* 93 (1) (1993) 154–161.
- 768 [70] J. P. Leão Neto, J. H. Lopes, G. T. Silva, Core-shell particles that are
769 unresponsive to acoustic radiation force, *Phys. Rev. Applied* 6 (2016)
770 024025.
- 771 [71] T. Nieminen, H. Rubinsztein-Dunlop, N. Heckenberg, Multipole ex-
772 pansion of strongly focussed laser beams, *J. Quant. Spect. and Rad.*
773 *Transf.* 79 (2003) 1005–1017.
- 774 [72] T. Nieminen, V. Loke, A. Stilgoe, G. Knner, A. Branczyk, N. Hecken-
775 berg, Optical tweezers computational toolbox, *J. Opt. A : Pure Appl.*
776 *Opt.* 9 (2007) 196–203.
- 777 [73] L. Gor’kov, On the forces acting on a small particle in an acoustic field
778 in an ideal fluid, *Sov. Phys. Dokl* 6 (1962) 773–775.
- 779 [74] T. A. Nieminen, G. Knöner, N. Heckenberg, H. Rubinsztein-Dunlop,
780 *Physics of optical tweezers*, *Methods Cell Biol.* 82 (2007) 207–236.

- 781 [75] Y. Harada, T. Asakura, Radiation forces on a dielectric sphere in the
782 rayleigh scattering regime, *Optics Comm.* 124 (1996) 529–541.
- 783 [76] P. Chaumet, M. Nieto-Vesperinas, Coupled dipole method determina-
784 tion of the electromagnetic force on a particle over a flat dielectric
785 substrate, *Phys. Rev. B* 61 (20) (2000) 14119–14127.
- 786 [77] A. Ashkin, J. Dziedzic, T. Yamane, Optical trapping and manipulation
787 of single cells using infrared laser beams, *Nature* 330 (6150) (1987) 769–
788 771.
- 789 [78] D. G. Grier, A revolution in optical manipulation, *Nature* 424 (6950)
790 (2003) 810–816.
- 791 [79] K. Dholakia, T. Čižmár, Shaping the future of manipulation, *Nature*
792 *Photonics* 5 (6) (2011) 335–342.
- 793 [80] A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, C. Cohen-
794 Tannoudji, Laser cooling below the one-photon recoil energy by
795 velocity-selective coherent population trapping, *Phys. Rev. Lett.* 61 (7)
796 (1988) 826.
- 797 [81] M. Padgett, S. Barnett, R. Loudon, The angular momentum of light
798 inside a dielectric, *J. Mod. Opt.* 50 (10) (2003) 1555–1562.
- 799 [82] J. Friend, L. Y. Yeo, Microscale acoustofluidics: Microfluidics driven
800 via acoustics and ultrasonics, *Reviews of Modern Physics* 83 (2) (2011)
801 647.
- 802 [83] M. Evander, J. Nilsson, Acoustofluidics 20: applications in acoustic
803 trapping, *Lab on a chip* 12 (22) (2012) 4667–4676.
- 804 [84] S. Kang, C. Yeh, Potential-well model in acoustic tweezers, *IEEE Ul-*
805 *trasonics* 57 (6) (2010) 1451–1459.
- 806 [85] C. R. Courtney, B. W. Drinkwater, C. E. Demore, S. Cochran, A. Gri-
807 nenko, P. Wilcox, Dexterous manipulation of microparticles using
808 bessel-function acoustic pressure fields, *App. Phys. Lett.* 102 (12)
809 (2013) 123508.

- 810 [86] J. Arlt, M. J. Padgett, Generation of a beam with a dark focus sur-
811 rounded by regions of higher intensity:the optical bottle beam, *Opt.*
812 *Lett.* 25 (4) (2000) 191–193.
- 813 [87] C.-H. Chen, P.-T. Tai, W.-F. Hsieh, Bottle beam from a bare laser for
814 single-beam trapping, *Appl. Opt.* 43 (32) (2004) 6001–6006.
- 815 [88] J. Nye, M. Berry, Dislocations in wave trains, *Proc. R. Soc. London*
816 336 (1974) 165–190.
- 817 [89] M. Soskin, M. Vasnetsov, Singular optics, *Progress in optics* 42 (2001)
818 219–276.
- 819 [90] G. Gibson, J. Courtial, M. J. Padgett, M. Vasnetsov, V. Pasko, S. M.
820 Barnett, S. Franke-Arnold, Free-space information transfer using light
821 beams carrying orbital angular momentum, *Optics Express* 12 (22)
822 (2004) 5448–5456.
- 823 [91] M. R. Dennis, K. O’Holleran, M. J. Padgett, Singular optics: optical
824 vortices and polarization singularities, *Progress in Optics* 53 (2009)
825 293–363.
- 826 [92] C. Cain, S. Umemura, Concentric-ring and sector-vortex phased-array
827 applicators for ultrasound hyperthermia, *IEEE Transactions on Mi-*
828 *crowave Theory and Techniques* 34 (5) (1986) 542–551.
- 829 [93] C. Tamm, C. O. Weiss, Bistability and optical switching of spatial
830 patterns in a laser, *J. Opt. Soc. Am. B* 7 (6) (1990) 1034–1038.
- 831 [94] B. T. Hefner, P. L. Marston, An acoustical helicoidal wave transducer
832 with applications for the alignment of ultrasonic and underwater sys-
833 tems, *J. Acous. Soc. Am.* 106 (6) (1999) 3313–3316.
- 834 [95] J.-L. Thomas, R. Marchiano, Pseudo angular momentum and topolog-
835 ical charge conservation for nonlinear acoustical vortices, *Phys. Rev.*
836 *Lett.* 91 (24) (2003) 1–4.
- 837 [96] R. Marchiano, J.-L. Thomas, Synthesis and analysis of linear and non-
838 linear acoustical vortices, *Phys. Rev. E.* 71 (2005) 1–11.
- 839 [97] R. Marchiano, J.-L. Thomas, Doing arithmetic with nonlinear acoustic
840 vortices, *Phys. Rev. Lett.* 101 (2008) 1–4.

- 841 [98] T. Brunet, J.-L. Thomas, R. Marchiano, F. Coulouvrat, Experimental
842 observation of azimuthal shock waves on nonlinear acoustical vortices,
843 *New J. Phys.* 11 (2009) 013002.
- 844 [99] T. Brunet, J.-L. Thomas, R. Marchiano, Transverse shift of helical
845 beams and subdiffraction imaging, *Phys. Rev. Lett.* 105 (3) (2010)
846 034301.
- 847 [100] B. W. Drinkwater, Dynamic-field devices for the ultrasonic manipula-
848 tion of microparticles, *Lab on a Chip* 16 (2016) 2360–2375.